

# Dark energy from dark radiation in strongly coupled cosmologies with no fine tuning.

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**ABSTRACT:** A dual component made of non-relativistic particles and a scalar field, exchanging energy, naturally falls onto an attractor solution, making them a (sub)dominant part of the cosmic energy during the radiation dominated era, provided that the constant  $\beta$ , measuring the coupling, is strong enough. The density parameters of both components are then constant, as they expand as  $a^{-4}$ . If the field energy is then prevalently kinetic, as is expected, its energy is exactly half of the pressureless component; the dual component as a whole, then, has a density parameter  $\Omega_{cd} = 3/4\beta^2$  (e.g., for  $\beta \simeq 2.5$ ,  $\Omega_{cd} \simeq 0.1$ , in accordance with Dark Radiation expectations). The stationary evolution can only be broken by the rising of other component(s), expanding as  $a^{-3}$ . In a realistic scenario, this happens when  $z \sim 3\text{--}5 \times 10^3$ . When such extra component(s) become(s) dominant, the densities of the dual components also rise above radiation. The scalar field behavior can be easily tuned to fit Dark Energy data, while the coupled DM density parameter becomes  $\mathcal{O}(10^{-3})$ . This model however requires that, at present, two different DM components exist. The one responsible for the break of the stationary regime could be made, e.g., by thermally distributed particles with mass even  $\gg 1\text{--}2$  keV (or non-thermal particles with analogous average speed) so accounting for the size of observed galactic cores; in fact, a fair amount of small scale objects is however produced by fluctuation re-generated by the coupled DM component, in spite of its small density parameter, after the warm component has become non-relativistic.

**KEYWORDS:** cosmology: theory, dark matter, dark energy, gravitation; methods: numerical..

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## 1. Introduction

Cosmological models supported by data are affected by a number of fine tuning and coincidence paradoxes. Among them, we have the problems concerning Dark Energy (DE), namely if its state parameter  $w \equiv -1$  : (i) Why are we living in the only era when DE and matter have comparable densities? (ii) Why density inhomogeneities are normalized so that DE allows fluctuations to reach the non–linear regime, then stopping any further density evolution?

Models trying to ease these paradoxes were proposed in the last few years, often based on the idea that DE is a scalar field [1, 2, 3], possibly coupled to Dark Matter (DM) [4]. Current data, however, do not provide sufficient discrimination, so that no such model really appears statistically favored in respect to  $\Lambda$ CDM (see, e.g., [5]). Accordingly, rather than suggesting models, recent work has focused on planning measures allowing us to discriminate between a DE state equation  $w(a) \equiv -1$  and other behaviors, as the forthcoming Euclid mission<sup>1</sup> [6]

In this paper, in a sense, we partially go back to the older approach, by suggesting a class of cosmological models, based on the assumption that DE is a scalar field  $\phi$ . In describing them, however, no peculiar self–interaction potential  $V(\phi)$  is selected, as the features we point out are (almost) independent from it, so that changing  $V(\phi)$  has a modest impact on our findings. Accordingly, while we expect that the shape of  $V(\phi)$  can

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<sup>1</sup><http://www.euclid-ec.org>

be hardly discriminated from  $\Lambda$ CDM through CMB and related data fits (see, e.g., [7]), we suggest specific predictions possibly discriminating this class of models from different cosmologies.

The basic point made in this paper is that a large fraction (typically 1/3) of Dark Radiation (DR) is a scalar field  $\phi$ , later turning into Dark Energy. Big-Bang Nucleosynthesis (BBN) is consistent with observable nuclide abundances in the presence of an extra radiative component consistent with  $\Delta N_{\text{eff}} < 1.26$  neutrino species [9], and this constraint can be further softened in the presence of primeval lepton anti-lepton asymmetry. CMB data are slightly more constraining, as the best fit of WMAP and related data yields  $\Delta N_{\text{eff}} = 0.85 \pm 0.62$  (2 standard deviations) [10]. A large deal of work has therefore been dedicated to the nature of DR, also outlining that the amount of DR could be quite different in BBN and CMB data [11].

The price to pay for DR turning into DE is that two kinds of Dark Matter (DM) must exist, that we shall dub  $DM_{\text{cou}}$  and  $DM_{\text{unc}}$ . The former one ( $DM_{\text{cou}}$ ) is coupled to the scalar field  $\phi$ . In the radiative era it is the rest of DR; in the Newtonian limit, it has ordinary gravitational interactions with any other cosmic components, but its fluctuations feel a much stronger self gravity, while its dynamics has further specific modifications [12]. Its density parameter, in the present epoch, shall lay in the per mil range, but its peculiar gravitational behavior can give it an important role in shaping a large deal of today's observables. The latter DM kind ( $DM_{\text{unc}}$ ), expected to have an ordinary gravitational behavior, is then needed to break a primeval stationary condition. In principle, quite a few discriminatory predictions may follow from the presence of the  $DM_{\text{cou}}$  component, coupled to DE.

Although the mechanism turning DR into DE is the basic point of this work, we shall devote a specific Section to discuss possible scenarios allowed by the presence of 2 DM kinds. Rather than a complication, this appears as an “opportunity”. In particular, we shall discuss the case of  $DM_{\text{unc}}$  being warm; the simultaneous presence of  $DM_{\text{cou}}$  might then ease a number of cosmological problems. Among them, the small halo deficit and the size of the plateau in galaxy cores, when comparing observations to simulations.

All that is obtained by pushing to extreme consequences the idea of DE–DM coupling, as previously suggested in [4]. The coupling intensity considered is however much greater than any previous analysis, and consistency with data can be recovered thanks to the fact that  $DM_{\text{cou}}$  is a minor component of DM.

The plan of the paper is as follows: In the next Section we shall discuss how to treat coupled DM and DE if the DE state equation  $w(a)$  is assigned, while the DE self-interaction potential  $V(\phi)$  is unknown. The equation of motion to be solved is then simpler than the usual Klein–Gordon equation, as the problem becomes first order, the real unknown being  $\phi_1 \equiv \dot{\phi}$ , while we do not need to know  $\phi$ . In Section 3, we then consider the  $\phi_1$  equation and the equation ruling  $DM_{\text{cou}}$  density in the radiative era, finding a self-consistent solution, which allows constant density parameters for  $DM_{\text{cou}}$  and  $\phi$ -field, provided that the coupling strength is large enough. In Section 4 we verify this solution to be an attractor, and that we converge on it if starting from any initial condition. In Section 5 we briefly discuss the form of the effective Lagrangian yielding the equation of motion. Section 6 is

then devoted to debating what happens if another non-relativistic component finalizes the radiative expansion. In Section 7 we then show that, rather than a complication, the two DM components are an opportunity, in particular if we assume  $DM_{unc}$  to be WDM. A Discussion Section concludes the paper.

## 2. CDM–DE coupling

The possibility that CDM and DE are coupled have been considered by several authors [4, 13, 14]. As a matter of fact, while the stress–energy tensors of CDM and DE,  $T_{(c)\mu\nu}$  and  $T_{(d)\mu\nu}$  respectively, surely fulfill the pseudo-conservation equation

$$T_{(c)\mu;\nu}^{\nu} + T_{(d)\mu;\nu}^{\nu} = 0 , \quad (2.1)$$

there is no direct evidence that the two equations

$$T_{(c)\mu;\nu}^{\nu} = 0, \quad T_{(d)\mu;\nu}^{\nu} = 0 \quad (2.2)$$

are separately satisfied. The r.h.s. of the above equations can then be replaced, in a covariant way, by a term yielding a leakage of energy from DE to CDM or viceversa.

Here we shall consider the option yielding

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} + a^2 V'_{\phi}(\phi) = C\rho_c a^2 , \quad \dot{\rho}_c + 3\frac{\dot{a}}{a}\rho_c = -C\dot{\phi}\rho_c . \quad (2.3)$$

Here  $\rho_c$  is the density of CDM,  $\phi$  is a scalar field self-interacting through the potential  $V(\phi)$  and accounting for DE, while

$$C = \frac{b}{m_p} = \sqrt{\frac{16\pi}{3}} \frac{\beta}{m_p} > 0 \quad (2.4)$$

accounts for energy transfer from CDM to DE. Taking a constant  $\beta$  is an extra assumption we shall make here for simplicity. Here we also assume a FRW metric reading

$$ds^2 = a^2(\tau)(d\tau^2 - d\ell^2) , \quad (2.5)$$

so that  $a$  is the scale factor,  $d\ell$  is the spatial line element, while differentiations are made in respect to the conformal time  $\tau$ .

The option (2.4) has a peculiar significance, as it allows models where DE is always a significant component of the Universe, being fed energy by the CDM component which, accordingly, dilutes more rapidly than  $a^{-3}$ .

Let us then reconsider the eqs. (2.3) when the potential  $V(\phi)$  is unknown, while we know that the DE state equation is a suitable  $w(a)$ . It must however be

$$w = \frac{\dot{\phi}^2/2a^2 - V}{\dot{\phi}^2/2a^2 + V} , \quad \text{i.e.} \quad V = \frac{\dot{\phi}^2}{2} \frac{1-w}{a^2(1+w)} , \quad (2.6)$$

and  $a^2 V'_{\phi}$ , in eq. (2.3), can be directly evaluated by considering how the two factors in the last expression depend on  $\tau$ , then associating such dependence with the  $\tau$  dependence of  $\phi$ .

Let us then notice that:

$$\frac{d}{d\phi} \frac{\dot{\phi}^2}{2} = \frac{d}{d\dot{\phi}} \frac{\dot{\phi}^2}{2} \times \frac{d\dot{\phi}}{d\tau} \times \frac{d\tau}{d\phi} = \ddot{\phi} , \quad (2.7)$$

$$\frac{d}{da} \left( \frac{1}{a^2} \frac{1-w}{1+w} \right) = -\frac{2}{a^3} \left[ \frac{1-w}{1+w} + \frac{dw}{da} \frac{a}{(1+w)^2} \right] \quad (2.8)$$

so that

$$a^2 V'_\phi = \ddot{\phi} \frac{1-w}{1+w} - \frac{\dot{a}}{a} \dot{\phi} \left[ \frac{1-w}{1+w} + \frac{dw}{da} \frac{a}{(1+w)^2} \right] . \quad (2.9)$$

Let then

$$\tilde{W} = \frac{1}{2} \left( 1 + 3w - \frac{a}{1+w} \frac{dw}{da} \right) \quad (2.10)$$

in order that the system of equations (2.3) becomes

$$\dot{\phi}_1 + \tilde{W} \frac{\dot{a}}{a} \phi_1 = \frac{1+w}{2} C \rho_c a^2 , \quad \dot{\rho}_c + 3 \frac{\dot{a}}{a} \rho_c = -C \phi_1 \rho_c . \quad (2.11)$$

Here we set  $\phi_1 = \dot{\phi}$  to outline that the former equation has become first order.

As a matter of fact, it seems more realistic that data allows us to know  $w(a)$ , rather than the potential  $V(\phi)$ . Should we know  $w(a)$  and wish to interpret the  $w(a)$  dependence as due to the evolution of a scalar field, we need to integrate just eqs. (2.11), together with the Friedman equation. The scalar field contribution to its source term is then the total energy density for DE,

$$\rho_d = \frac{\phi_1^2}{a^2(1+w)} , \quad (2.12)$$

as also the potential contribution is derived from  $\phi_1$  and  $w(a)$ , thought eq. (2.6). Apparently, therefore, we need not recovering the un-differentiated  $\phi$  behavior.

This however assumes that we know DE and  $DM_{cou}$  to be coupled, that the coupling is constant, and the coupling constant has a specific value  $C = 4(\pi/3)^{1/2} \beta / m_p$ .

It is premature to discuss here observational strategies. A natural start point, however, amounts to assuming no coupling. The apparent DE state parameter, measured from from the expansion rate, would then be

$$w_{eff}(a) = w(a) / [1 + \xi(a)] \quad (2.13)$$

with

$$\xi(a) = [g(\phi)/g(\phi_0) - 1] \times \rho_{0c} / (\rho_d a^3) . \quad (2.14)$$

Here  $g(\phi) \propto \rho_c a^3$  tells us the deviation of  $\rho_c$  from the ordinary  $a^{-3}$  scaling; the suffix <sub>0</sub> refers anywhere to today's quantities (see [8] for a detailed discussion).

Notice that, in order to pass from  $w(a)$  to  $w_{eff}(a)$  or viceversa, we then need to know  $\phi$ , besides of  $\phi_1$ . However, to recover  $\phi$ , we do not need integrating an equation, but just a known function.

We shall not delve here into a possible more refined analysis, seeking the family of  $w_\beta(a)$  behaviors as the assumed coupling  $\beta$  varies.

Notice also that eq. (2.12) shows that  $\rho_d$  is positive definite only if  $w > -1$ : to explore the  $w < -1$  domain one needs the  $\phi$  field to have a suitable anomalous kinetic energy expression.

These equations, therefore, require no specific potential shape to be assigned, no background expansion regime to be assumed, while the very  $w(a)$  behavior is generic, although it must be  $w(a) > -1$ .

### 3. Coupled DE in the radiative era

Let us now consider the system (2.11) when the background expansion is supposed to be radiative. We shall make the further assumption that

$$\tilde{W} = \frac{1}{2}(1 + 3w) , \quad (3.1)$$

with  $w = \text{const.}$ , as is reasonable when  $a \rightarrow 0$ . This assumption is however unessential and only allows us to simplify the analytical treatment.

Let us also remind that, in the radiative era, it is  $a \propto \tau$ , so that  $\dot{a}/a = 1/\tau$ . Accordingly, from Friedmann equations we obtain that

$$\frac{8\pi}{3m_p^2} \rho a^2 \tau^2 = 1 , \quad (3.2)$$

$\rho$  being the background energy density and  $m_p$  the Planck mass. The latter eq. (2.11) has then the formal integral

$$\rho_c = \rho_{i,c} \left( \frac{a_i}{a} \right)^3 \exp \left( -C \int_{\tau_i}^{\tau} d\tau \phi_1 \right) , \quad (3.3)$$

$\tau_i$  being a reference time when CDM density is  $\rho_{c,i}$  and the scale factor is  $a_i = a(\tau_i)$ . If this expression for  $\rho_c$  is then replaced in the former eq. (2.11), we have a first order transcendental differential equation whose unknown is  $\phi_1$ . It seems hard to find a generic analytic integral of this equation.

There is however a peculiar case, allowing integration. Let us make the ansatz that

$$\phi_1 = \alpha \frac{m_p}{\tau} . \quad (3.4)$$

Taking eq. (2.4) into account, we have then that

$$-C \int_{\tau_i}^{\tau} d\tau \phi_1 = \ln \left( \frac{\tau_i}{\tau} \right)^{\alpha b} , \quad (3.5)$$

so that

$$\rho_c = \rho_{i,c} \left( \frac{a_i}{a} \right)^{3+\alpha b} \quad (3.6)$$

and, by replacing the expression (3.4) in the former eq. (2.11), we obtain

$$(\tilde{W} - 1) \alpha \frac{m_p}{a^2 \tau^2} = \frac{1+w}{2} \frac{b}{m_p} \rho_{rc} \left( \frac{a_r}{a} \right)^{3+\alpha b} . \quad (3.7)$$

In order that the two sides scale with  $a$  in the same way, it must then be  $\alpha b = 1$  and the DM density shall scale with  $a^{-4}$ . The fact that  $\rho_c$  dilutes more rapidly than  $\propto a^{-3}$  does not come as a surprise, as there is a continuous leakage of energy from it to the  $\phi$  field. The fact that it dilutes exactly as  $a^{-4}$ , instead, is a consequence of the ansatz (3.4).

Equation (3.7) can then be put in the form

$$\frac{1}{\beta^2} \frac{\tilde{W} - 1}{1 + w} = \frac{8\pi}{3m_p^2} \rho_c a^2 \tau^2 \equiv \Omega_c , \quad (3.8)$$

owing to eq. (3.2). Here  $\Omega_c = \rho_c/\rho$  is the (constant) density parameter of DM, during radiation era. In order that the ansatz (3.4) is allowed,  $\Omega_c$  ought to have the value given by this equation.

Also the energy density  $\rho_d$  of the DE field  $\phi$  scales with  $a^{-4}$ . In fact, owing to eq. (2.12),

$$\rho_d = \frac{\alpha^2 m_p^2}{a^2 \tau^2} \frac{1}{1 + w} , \quad (3.9)$$

and, using again eq. (3.2), we obtain the constant density parameter of DE

$$\Omega_d = \frac{1}{2\beta^2(1 + w)} \quad (3.10)$$

showing also that

$$\frac{\Omega_c}{\Omega_d} = 2(\tilde{W} - 1) = 3w - 1 . \quad (3.11)$$

Accordingly, the whole framework is consistent only if  $w > 1/3$ . This means that the energy density of the kinetic part of the  $\phi$  field should be dominant in respect to the potential part. In the specific case  $w \simeq 1$ , holding for  $\phi_1^2 \gg 2a^2 V$ , we therefore expect that it is constantly

$$\Omega_c \simeq 2 \Omega_d \quad (3.12)$$

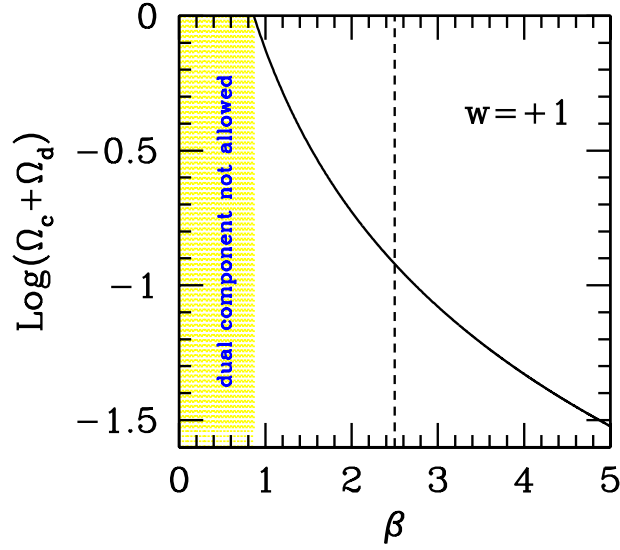
during such expansion regime.

Altogether, these computations show that, during a radiative expansion, we can have two coupled components also expanding  $\propto a^{-4}$  although their state equations are  $w_c = 0$  and  $w > 1/3$ . The former component is made of non-relativistic particles, the latter is a self-interacting scalar field. A possible option is that the individual CDM particle masses decrease in time, as a consequence of their interaction with  $\phi$ . The two dark components interact with a strength measured by the dimensionless parameter  $\beta$ .

Owing to their behavior, these two components do not modify the radiative character of the expansion. However, we might prefer that ordinary radiation is the dominant component during this period. This requires a strong coupling between the components  $\beta \gg 1$ , as it should however be  $w \leq 1$ . In turn, the ratio between the density of DM and DE is  $\mathcal{O}(1)$ .

In the specific case  $w \simeq 1$ , however reasonable in the very early Universe, we then have  $\Omega_d \beta^2 = 1/4$  and

$$(\Omega_c + \Omega_d) \beta^2 = 3/4 . \quad (3.13)$$



**Figure 1:** Density parameter of the dual radiative component vs. the coupling parameter  $\beta$

In an early epoch it is fair to assume a total density parameter  $\Omega_t = 1$ . Then, requiring  $\Omega_c + \Omega_d < 1$  yields

$$\beta > \sqrt{3}/2 = 0.866 . \quad (3.14)$$

A solution with  $w \simeq 1$  and  $\beta^2 \simeq 3/4$ , although self-consistent, seems unreasonable. In fact, then  $\Omega_c + \Omega_d \simeq 1$  and the ordinary radiation component should vanish.

Solution with  $\beta^2 < 3/4$  require  $w > 1$  to allow constant  $\Omega_{c,d}$ . If  $w > 1$  is excluded, when  $\beta^2 < 3/4$  there exist no solution with constant  $\Omega_{c,d}$ , i.e., the CDM and  $\phi$  field contributions to the overall density become increasingly small when  $a$  tends to zero.

In Figure 1 we plot the density parameter of the dual (DM+DE) component vs. assumed  $\beta$  values.

The dual component gives place to a natural form of DR. In general, we can gauge its significance through the number of extra neutrino species

$$\Delta N_{off} = \frac{\rho_{DR}}{\frac{\pi^2}{30} \frac{7}{4} \left(\frac{4}{11}\right)^{4/3} T^4} \quad (3.15)$$

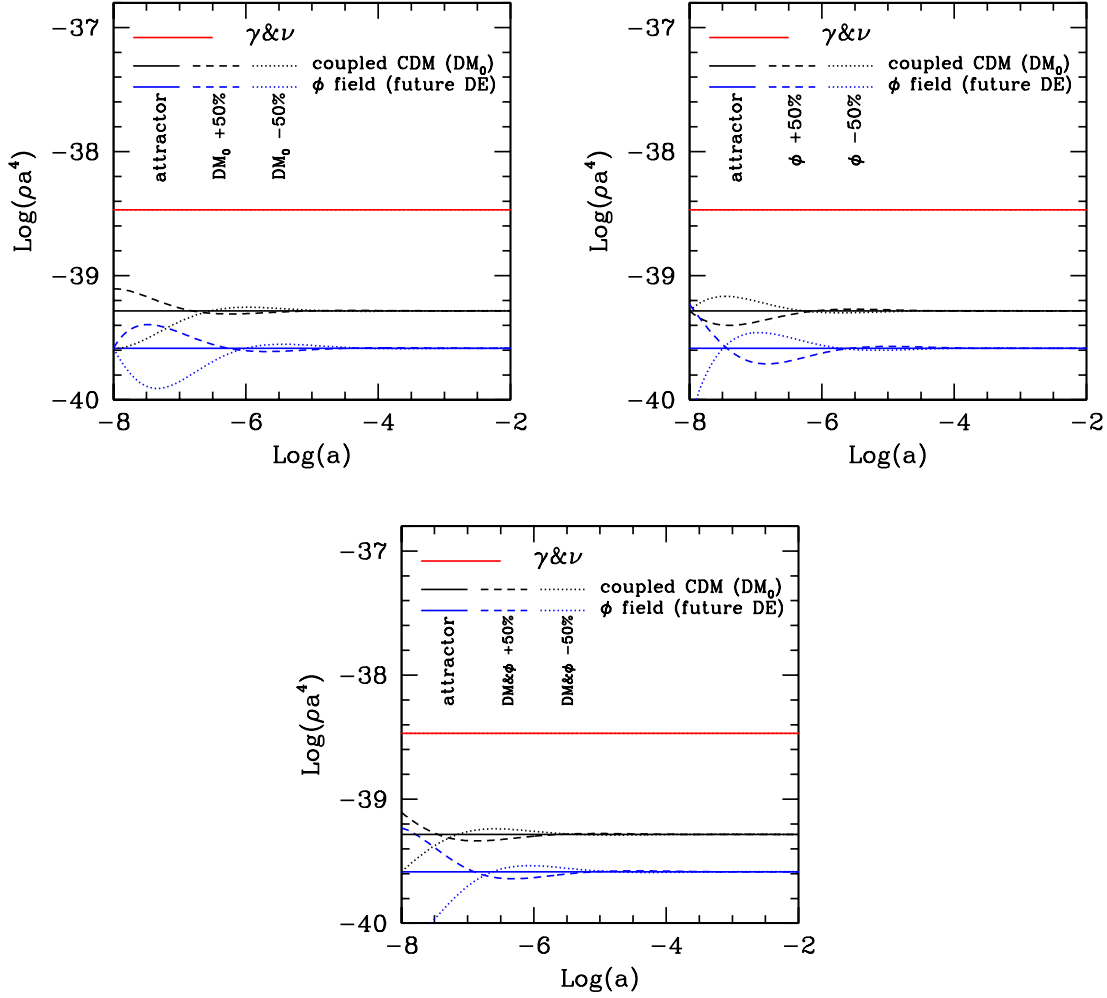
(here  $\rho_{DR}$  is the dual-component density and  $T$  is photon temperature). It is then

$$\Delta N_{off} = \frac{(8/7)(11/4)^{4/3} + 3}{(4/3)\beta^2 - 1} \simeq \frac{7.4032}{(4/3)\beta^2 - 1} \quad (3.16)$$

and  $\beta = 1.5$  (2.5) yields  $\sim 3.7$  (1.1) extra species.

Let us recall that BBN prescribes  $\Delta N_{off} < 1.26$ , in standard theories, or  $\Delta N_{off} < 2.56$  when a non-vanishing chemical potential is allowed for neutrinos [9]. The overall density of radiative components also sets the equality redshift. CMB and related data analysis then allow us to fix the equality, so requiring  $\Delta N_{off} \simeq 0.85 \pm 0.62$  or similar figures [10], although the choice of priors risks to affect the final estimate [15]. All above limits are at





**Figure 2:** Stability of solutions. We start from modified initial values, showing that we soon reconverge on the “attractor” solution. More specifically, we shifted by  $\pm 50\%$ , respectively:  $\rho_c$ ,  $\phi_1$ , both of them; the last option is equivalent to setting a “modified”  $\beta$  value. The solid lines are the “attractor” solution. Dashed and dotted lines show the gradual recovery of it. In these Figures  $\beta = 2.5$ .

95% confidence level. Let us also recall that models can be easily built, where the DR density at equality exceeds the one at BBN. The smallest coupling consistent with all above limits is however around  $\beta = 2$ . Through this paper, only the case  $\beta = 2.5$  will be however considered.

#### 4. Stability

Solutions with  $w = 1$  and

$$\Omega_c = 2\Omega_d = 1/2\beta^2 \quad (4.1)$$

are however stable. If the initial values of  $\Omega_c$ ,  $\phi$ , or  $\beta$  do not fulfil the above relation, and the expansion regime is radiative, the condition (4.1) is soon restored.

In Figure 2 we show this in 3 cases: (i) If we set an initial value of  $\rho_c$  in excess by 50 %. (ii) If we set an initial value of  $\phi_1$  in excess by 50 %. (iii) If both shifts are simultaneously performed; this is equivalent to having a “wrong” initial  $\beta$ .

The results in Figure 2 are obtained by numerically integrating the set of differential equations (2.3) plus the Friedmann equation

$$(\dot{a}/a)^2 = 8\pi\rho/3m_p^2, \quad (4.2)$$

$\rho$  being the total background density. All quantities are expressed in MeV; in particular the units for the conformal time  $\tau$  (not shown) are  $\text{MeV}^{-1}$ .

The above numerical output can be analytically understood. Let us consider, e.g., that the density of the DE component has a value different from what eq. (3.10) requires, i.e. that

$$\phi_1 = \phi_{1,o} + \delta \quad \text{and} \quad \phi_{1,o} = \alpha m_p / \tau \quad (4.3)$$

with  $\delta$  positive or negative. Assuming that  $\rho_c$  is unmodified, eq. (2.11) tell us that  $\delta$  must fulfill the equation

$$\dot{\delta} + (1 + \epsilon)\delta/\tau = 0 \quad (4.4)$$

with  $\epsilon = \tilde{W} - 1 > 1$  (for  $w > 1/3$ ). Then,  $\delta \propto \tau^{-(1+\epsilon)}$  and the ratio

$$|\delta|/\phi_1 \propto \tau^{-\epsilon} \quad (4.5)$$

necessarily decreases with time. As a matter of fact, however,  $\phi_1$  appears also in the differential equation setting  $\dot{\rho}_c$ , while the impact on Friedmann equation is small if we assume a large  $\beta$  yielding small  $\Omega_{c,d}$  (plots are for  $\beta = 2.5$ ). A greater  $\phi_1$  then yields a  $\rho_c$  decrease faster than  $a^{-4}$ . In turn this corresponds to a decreased energy leakage towards the scalar field, so that  $|\delta|$  decline is accelerated, in respect to (4.5). Figure 2 however shows that some bounces occur before the “attractor” solution is recovered.

Analogous arguments can be put forward for the cases (ii) and (iii). For the sake of completeness, let us also outline that solutions are tendentially stable also for constant  $w < 1$ . However, if initial conditions violate eqs. (3.10)–(3.11), the recovery of the attractor solution takes an increasingly longer time as  $w$  is farther from unity.

## 5. A Lagrangian approach

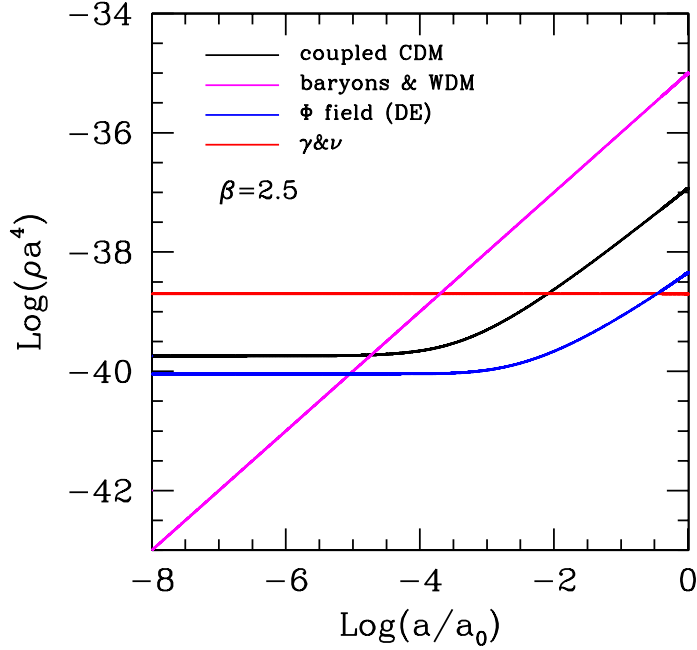
The function  $g(\phi)$  needed to define  $\xi$  in eq. (2.13) also enters in the Lagrangian coupling between the scalar  $\phi$  field and a supposed spinor field  $\psi$ , yielding  $DM_{cou}$ .

According to [8], the general expression of the effective interaction Lagrangian reads

$$\mathcal{L} = \mu g(\phi) \bar{\psi} \psi, \quad (5.1)$$

$\mu$  being a factor with the dimensions of a mass. Owing to eq. (3.3), then, it should be

$$g(\phi) = \exp \left( -b \int_{\tau_p}^{\tau} d\tau \phi_1 / m_p \right) \quad (5.2)$$



**Figure 3:** Density evolutions when a non-relativistic component becomes dominant at  $z \simeq 5 \times 10^3$ . Both  $\text{DM}_{\text{cou}}$  and DE densities then overcome radiation, but keep steadily below  $\text{DM}_{\text{unc}}$ . We assumed here  $\beta = 2.5$ , so that  $\Omega_c + \Omega_d \equiv 0.12$  in the radiative era; this corresponds to  $\Delta N_{\text{off}} = 1.009$ , as Dark Radiation, i.e. to  $\simeq 1$  extra neutrino species.

so that, owing to the ansatz (3.4),

$$\mathcal{L} = \mu \frac{\tau_p}{\tau} \bar{\psi} \psi = b \frac{\mu}{m_p^2} \dot{\phi} \bar{\psi} \psi = \Gamma \dot{\phi} \bar{\psi} \psi, \quad (5.3)$$

with a constant  $\Gamma$  whose dimensions are  $m^{-1}$ , or

$$\mathcal{L} = \frac{\mu T}{m_p} \bar{\psi} \psi. \quad (5.4)$$

This makes clear that the very interaction Lagrangian also displays the role of variable-mass term, with a mass  $\propto \tau^{-1}$  or  $\propto T$ .

## 6. Exit from the radiative regime

The picture changes if the expansion is no longer radiative. In the physical world, there will be baryons, at least, whose density overcomes the density of the radiative components slightly below  $z \sim 10^3$ . If their abundance is consistent with BBN, they are surely not enough to produce an overall picture possibly close to observations. We shall therefore assume that another non-relativistic DM component exists so that its density summed to baryons matches the density of the radiative component at  $z = 5 \times 10^3$ . We have therefore

two DM components:  $DM_{cou}$  coupled to the field  $\phi$ ;  $DM_{unc}$  uncoupled; for the sake of simplicity, here baryons are included in  $DM_{unc}$ .

In Figure 3 we show how  $DM_{cou}$  and the DE field no longer scale as radiation, when the overall expansion ceases to be radiative. We assume that  $\beta = 2.5$ , so that  $\Delta N_{off} \simeq 1$ , and that the equation of state of DE keeps  $w = +1$  until today. Then,  $\phi$  does not contribute much to today's overall density while, by decreasing  $\beta$ , we could have  $DM_{unc}$  approaching the present baryon density, at most. Below, we shall further comment on  $DM_{unc}$  observable effects.

The situation changes radically if we include the option that the  $\phi$  field abandons the kinetic regime. Of course, this is necessary if we wish to identify it with DE, whose today's state equations approaches  $w \simeq -1$ . For most potentials considered in the literature, when  $\phi$  overcomes a suitable level, the potential energy becomes indeed dominant. The moment when the transition occurs depends on the potential assumed as well as on the initial contribution of the two coupled components to the cosmic budget.

As an example, we report here the expected behavior of  $w(z)$  when the potential is SUGRA [2] or RP [1]. More details on these plots are given in [14]. The Figures 4 show the shift of  $w$  from  $+1$  to  $\sim -1$ , with some potentials studied in the literature. Values  $\beta < \sqrt{3}/2$  are however taken in these Figures, while DE is coupled to all DM.

Our aim here, however, just amounts to showing that DE can achieve a density consistent with observations while  $DM_{cou}$ , on the contrary, yields a negligible contribution to the cosmic budget. This unavoidably requires that the energy density of the  $\phi$  field turns from (prevalently) kinetic to (prevalently) potential.

The only caution to be taken is avoiding a too fast  $w$  decrease, as the expression of  $\tilde{W}$  has a contribution from  $dw/da$ , which may become dangerously high.

In this work we take the following class of interpolatory functions

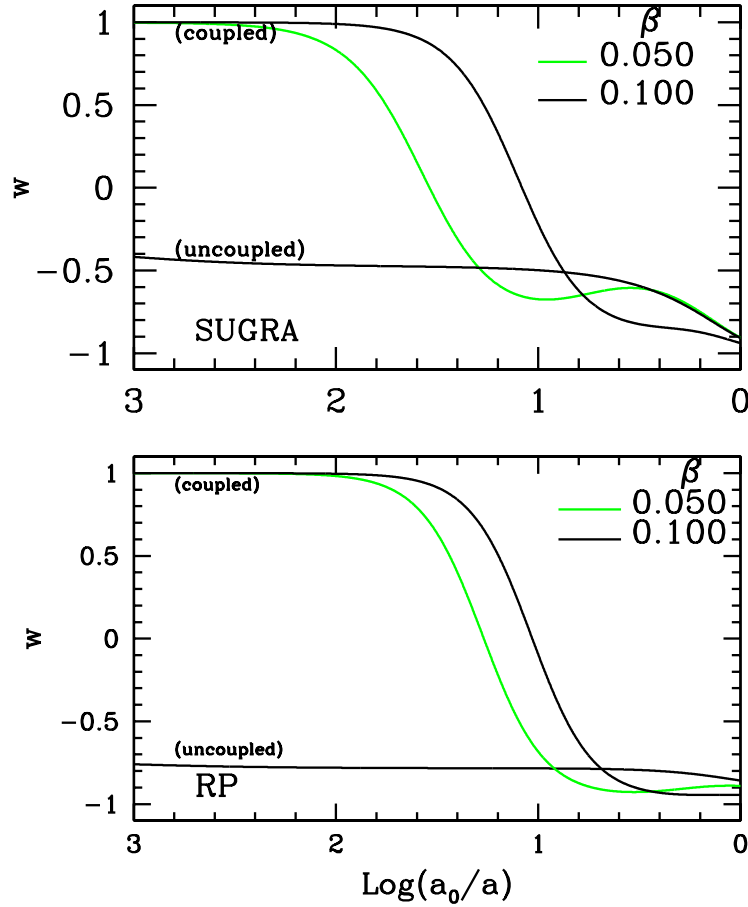
$$\begin{aligned} 1 + w &= (1 + w_+) \exp \left[ - \left( \frac{a}{a_1} \right)^\epsilon \right] & \text{for } a \leq a_1 \\ 1 + w &= \frac{1 + w_+}{e} \left( \frac{a_1}{a} \right)^\epsilon & \text{for } a > a_1 \end{aligned} \quad (6.1)$$

yielding

$$\begin{aligned} \frac{a}{1 + w} \frac{dw}{da} &= -\epsilon \left( \frac{a}{a_1} \right)^\epsilon \leq \epsilon & \text{for } a \leq a_1 \\ \frac{a}{1 + w} \frac{dw}{da} &= -\epsilon & \text{for } a > a_1 \end{aligned} \quad (6.2)$$

so that the  $\tilde{W}$  correction, in respect to  $1 + 3w$  is  $\epsilon$ , at most. Here  $w_+$  is the DE equation of state at large  $z$ . In principle,  $\epsilon$  is to be fixed so that the DE state equation is a suitable  $w_-$  at  $z = 0$ . Here we used  $\epsilon = 1.9$ . In Figure 5 we show the resulting  $w(a)$  behavior, when  $1 + z_1 = 1/a_1 = 12$ .

Notice that the  $w(a)$  behavior shown in the Figure is not so far from those obtained from some assigned potentials. In particular, the large value of  $\beta$  is balanced by the low  $DM_{cou}$  density which, however, is not arbitrary, as the initial  $DM_{cou}$  density parameter is dictated by  $\beta$  itself.

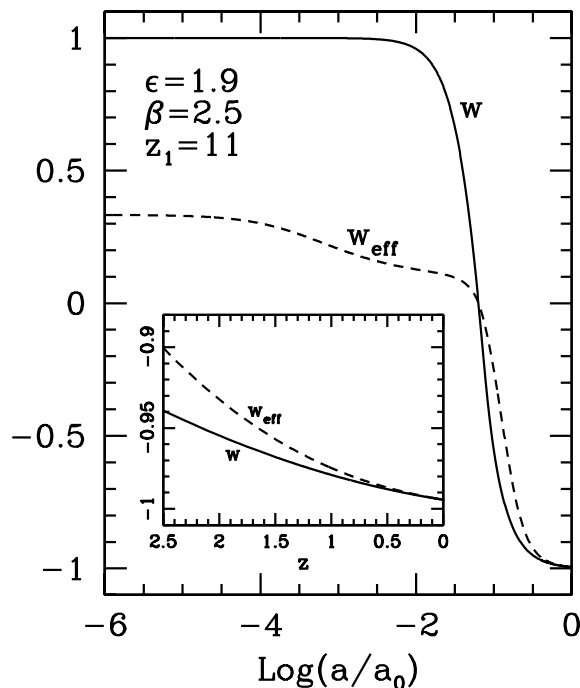


**Figure 4:** Scale dependence of the DE state parameter in SUGRA and RP models, in the presence and absence of energy transfer from a large CDM component. The main parameters of the models are  $\Omega_b = 0.046$ ,  $\Omega_c = 0.209$ ,  $H_0 = 73$  km/s/Mpc, while  $\Lambda = 0.1$  GeV for SUGRA and  $\Lambda = 10^{-5}$  GeV for RP. The values of the coupling  $\beta$  are shown in the frames.

In Figure 5 we also plot  $w_{eff}$ , defined in accordance with eq. (2.13). As expected from the required expansion rate, at large  $z$  it is  $w_{eff} = 1/3$ . When  $w$  falls down and intersects  $w_{eff}$ , it also starts to decrease. In the inner frame of the Figure, the low- $z$  behavior is magnified. Up to  $z \sim 1$  the scale dependence of  $w$  and  $w_{eff}$  will be distinguishable only through very refined experiments. Above  $z \sim 1$ , however, the difference becomes more relevant.

The interpolation (6.2) with  $z_1 = 11$  and  $\epsilon = 1.9$ , then yields the behavior of the different cosmic component shown in Figure 6. Let us specifically outline that the final DE density can be tuned rather easily to different values; more specifically, a greater (smaller)  $z_1$  yields a greater (smaller) today's DE density. On the contrary,  $DM_{cou}$  final density is substantially insensitive to the parameter choice.

Altogether, reproducing the observational densities requires a suitable set of model parameters, but no fine tuning is required: their tuning must be so precise as the parameter precision required. In the case of Figure 6, the present DE density is 3 times  $DM_{unc}$



**Figure 5:** Solid lines: DE state parameter during the kinetic–potential transition. Dashed lines: Effective state parameter, if hypothetical data are considered by supposing DE not to be coupled. The large  $\beta$  value balances the small  $DM_{cou}$  density, so that the  $w$  scale dependence is not so different, e.g., from a RP case with  $\beta = 0.1$  and all DM coupled (Figure 4). The inner panel shows the low- $z$  behavior, outlining that  $w$  and  $w_{eff}$  are not so different up to  $z \sim 1$ , again because of the small  $DM_{cou}$  density parameter.

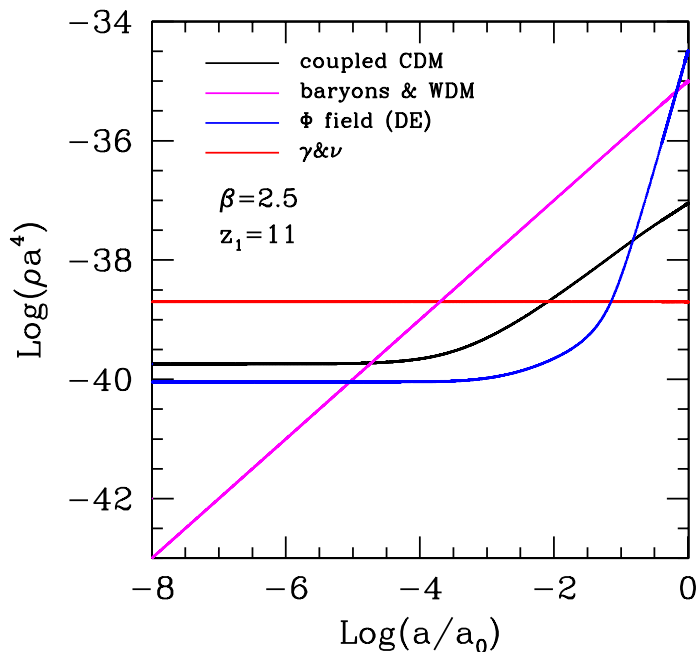
(including also baryons), while the final contribution of  $DM_{cou}$  to the density budget is in the per mil range.

## 7. Astrophysical context: flat galaxy cores and small halo deficit

If we then compare the cosmological picture proposed here with more standard scenarios, where DE is however a scalar field, the main difference is that here the  $\phi$  field has substantially contributed to the cosmic budget since ever.

On the contrary, multiple DM components have been considered by several authors (see, e.g., [16], [17],[18]), for various reasons. In this Section we shall debate the option that  $DM_{unc}$  is WDM, instead of CDM. In association with the presence of a second DM component, and owing to the peculiar features of  $DM_{cou}$ , this option seems particularly appealing.

Let us then first remind that, if a DM component is coupled to DE, the effective gravity it feels is modified. For baryonic matter, we have sophisticated tests allowing us to state that, on terrestrial and planetary scales, no coupling with DE exists [19]. No such test can be extended to DM and this is why quite a few options for DM–DE coupling were considered in the literature, and tested against cosmic data.



**Figure 6:** Density evolutions for a model as in Figure 2, with the DE state equation in Figure 5.

Observations, however, so nicely fitting most  $\Lambda$ CDM predictions, put stringent limits to the coupling  $\beta$  [20]. They can be eased when DM–DE coupling is considered in association with non-vanishing neutrino masses [14], and Mildly Mixed Coupled cosmologies were found to fit data slightly better than  $\Lambda$ CDM. The likelihood improvement, however, is not statistically significant and, even within this approach, a coupling range including  $\beta > 0.886$  is excluded. If two DM components exist, however, we are in a fully different context, that we may tentatively exploit to seek a solution to some inconsistencies of the  $\Lambda$ CDM model, on sub-galactic scales, put in evidence by N-body simulations, namely if DM is assumed to be “cold”.

A first difficulty concerns the amount of substructure in Milky Way sized haloes [23]. Models involving CDM overpredict their abundance by approximately one order of magnitude. A second issue concerns the density profiles of CDM haloes in simulations, exhibiting a cuspy behavior [24, 25], while the density profiles inferred from rotation curves suggest a core like structure [26]. A third issue concerns dwarf galaxies in large voids: recent studies [27] re-emphasized that they are overabundant.

It is known that replacing CDM with a “warmer” DM component, as a thermal relic of particles whose mass is  $\sim 2\text{--}3$  keV, yields better predictions. What is essential, however, is the streaming length of such component. Accordingly, it can also be replaced by particles of different mass, with a non-thermal distribution, but similar average velocity. However, there is a number of “thermal” candidates for such warm dark matter (WDM); among them, a sterile neutrino and a gravitino [28] find a reasonable motivation in particle theory

[29].

The long streaming length of such particles causes a strong suppression of the power spectrum on galactic and sub-galactic scales [30] and solves several above problems. In particular, the profiles of WDM haloes, similar to CDM haloes in the outer regions, flatten towards a constant value in the inner regions, as predicted in [31] and found in simulations [32].

However, the core size found is 30–50 pc, while the observed cores in dwarf galaxies are around the 1000 pc scale [33]. A dwarf galaxy core in this scale range would be produced by higher velocity particles, as those belonging to a thermal distribution if their mass is  $< 0.1$ – $0.3$  keV. Increasing the velocity, however, yields a greater streaming length, exceeding the size of these very dwarf galaxies, in the first place [34].

In view of these difficulties, the idea that WDM is accompanied by a smaller amount of CDM has already been put forward [16]. The WDM particle velocities could then be greater, while a low-mass population is however produced by CDM clustering. This suggestion was been put forward quite independently of any particle or cosmic model. In particular, assuming *ad hoc* a twofold dark matter component does not ease coincidence problems.

It is then clear that the model discussed in this paper could be adapted to meet the above requirement.  $DM_{unc}$  would be a kind of WDM, made of high-speed particles.  $DM_{cou}$ , then, would be responsible to create condensation sites on scales smaller than the  $DM_{unc}$  streaming length, after its derelativisation. Its role is similar to the CDM role in  $\Lambda$ CDM models, after recombination, when CDM fluctuations cause baryon accretion on scales where primary baryon fluctuations had been erased during recombination.

At large scale we then expect a standard primeval fluctuation spectrum, suitably balanced between  $DM_{cou}$  and  $DM_{unc}$ , although today's  $DM_{cou}$  contribution could be not so significant. Below the streaming length of  $DM_{unc}$ , however, only  $DM_{cou}$  fluctuation initially remain, to create the seeds for the low scale fluctuation spectrum, after  $DM_{unc}$  derelativization. The amplitude of the overall DM spectrum can then be expected to be a few times smaller below the WDM ( $DM_{unc}$ ) streaming length.

These qualitative considerations require a detailed quantitative confirm. It is not unreasonable, however, that “secondary”  $DM_{unc}$  fluctuations, although still yielding some low-mass structure, generate less low-mass haloes than a standard CDM spectrum. The evolution of the fluctuations in  $DM_{cou}$  needs however a direct inspection. It is known that, at the Newtonian level, its gravitational self-interaction force is enhanced by a factor  $1 + 4\beta^2/3$  ( $\sim 9$  for  $\beta \sim 2.5$ ). Therefore, after entering the non-linear regime,  $DM_{cou}$  fluctuations, more rapidly than fluctuations in other components, could evolve into collapsed objects; their expected features are not easily predictable without a detailed analysis and should be then compared with observations.

Analogous comments can be made for the size of the cores in low mass galaxies. Being mostly made of low-mass WDM they can be as large as required, while the galaxy population does exist thanks to the  $DM_{cou}$  spectral seeds.

To put these expectations in a quantitative form we need to study fluctuation evolution in detail.



## 8. Discussion

In this work we considered coupled DE theories, when the coupling constant  $\beta$  is large. The first finding is then that a dual component, made of non-relativistic particles and a scalar field, can be in equilibrium with the radiative components in the radiative era. The density parameters of the dual components have then a fixed ratio

$$\Omega_c/\Omega_d = 2, \quad \text{while} \quad (\Omega_c + \Omega_d)\beta^2 = 3/4 \quad (8.1)$$

(if the scalar field energy is prevalently kinetic) and such density parameters keep constant, as both dual components dilute  $\propto a^{-4}$  as the Universe expands. We dubbed them  $\text{DM}_{cou}$  and DE although, at this stage, there is no evidence of the latter being related to observational DE. Another important finding is that the dual component is stable: if we set initial conditions violating (8.1), the densities of  $\text{DM}_{cou}$  and DE change and the condition (8.1) is restored.

The dual component gives place to a natural form of DR. Here we find that

$$\Delta N_{eff} \simeq \frac{5.55}{\beta^2 - 0.75}, \quad (8.2)$$

so that  $\beta = 2.5$  yields  $\Delta N_{eff} \simeq 1$ . All plots in this paper are given for this case.

When the expansion regime ceases to be radiative, as the density of a different non-relativistic component ( $\propto a^{-3}$ ) overcomes the radiative component, also the densities of  $\text{DM}_{cou}$  and DE start to increase. This further DM component is dubbed  $\text{DM}_{unc}$ .  $\text{DM}_{cou}$  and DE densities, although increasing, however keep well below  $\text{DM}_{unc}$ , unless the energy of the scalar field shifts from kinetic to potential.

Let us then recall again that, when we try to fit background data to a model where DE is coupled to the whole DM and self-interacts through a standard potential (e.g., Ratra-Peebles or SUGRA), we find that the  $\phi$  field energy shifts from kinetic to potential at a redshift  $z_1 \sim 10\text{--}20$ . This is the range where the transition must occur, also in this case. The example shown in the Figures is for  $z_1 = 11$ , but similar proportions of DE are obtained also by slightly shifting  $z_1$  and the parameter  $\epsilon$ , simultaneously. Large shifts of such parameters are however not allowed and the epoch of the field transition from kinetic to potential is well constrained, quite independently of the freedom we still have to modify its detailed dynamics.

Let us finally stress that we describe a stationary high- $z$  situation, holding since the decoupling of the  $\text{DM}_{cou}$  component from the other particles and, possibly, even before this stage. The dynamics discussed here could originate from a phenomenological Lagrangian coupling/mass term

$$\mathcal{L} = \Gamma \dot{\phi} \bar{\psi} \psi \quad \text{or} \quad \mathcal{L} = \frac{\mu T}{m_p} \bar{\psi} \psi, \quad (8.3)$$

holding since then. Here  $\psi$  describes spinor particles, which are  $\text{DM}_{cou}$ . Should the validity of this Lagrangian extend until the end of the inflationary era, one wonders whether any relation exist between the DE  $\phi$ -field and the scalar field responsible for the inflationary process itself.

ACKNOWLEDGMENTS - We thanks Matteo Viel, Marino Mezzetti and Luca Amendola for useful discussions. S.A.B. acknowledges the support of CIFS though the contract n. 24/2010 and its extension Prot.n.2011/338bis .

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